

Exercise 53

Find the derivative of the function. Simplify where possible.

$$F(x) = x \sec^{-1}(x^3)$$

Solution

Use the product rule, the chain rule, and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned} \frac{dF}{dx} &= \frac{d}{dx}[x \sec^{-1}(x^3)] \\ &= \left[\frac{d}{dx}(x) \right] \sec^{-1}(x^3) + x \left[\frac{d}{dx} \sec^{-1}(x^3) \right] \\ &= (1) \sec^{-1}(x^3) + x \left[\frac{1}{(x^3)\sqrt{(x^3)^2 - 1}} \cdot \frac{d}{dx}(x^3) \right] \\ &= \sec^{-1}(x^3) + x \left[\frac{1}{x^3\sqrt{x^6 - 1}} \cdot (3x^2) \right] \\ &= \sec^{-1}(x^3) + \frac{3}{\sqrt{x^6 - 1}} \end{aligned}$$